Phil 240

A Difficult extra credit problem

In our discussion of *Language, Proof, and Logic* §3.6 ("Equivalent ways of saying things"), I presented the 'standard' or 'official' characterization of synonymous sentences, i.e. sentences that have the same meaning. This characterization was:

Standard synonymy:	Two sentences s_1 and s_2 are *synonymous* = In every arrangement of named blocks on the Tarski's World checkerboard, s_1 and s_2 have the same truth-value.

In other words: in every arrangement where s_1 is true, s_2 is also true; and in every arrangement where s_1 is false, s_2 is also false. (It may not be obvious, but this entails the converse, i.e., in every arrangement where s_2 is true, s_1 is also true; and in every arrangement where s_2 is false, s_1 is also false.)

But during our discussion, another characterization was offered:

Gallagher synonymy:	Two sentences s_1 and s_2 are *synonymous* = The set of all the sentences that follow from s_1 (call this set $\mathcal{S}_1$) is identical to the set of all the sentences that follow from s_2 (call this set $\mathcal{S}_2$).

In other words: $\mathcal{S}_1 = \mathcal{S}_2$.

The interesting question is: are these two definitions equivalent? In other words: is there a pair of sentences that are standard-synonymous but not Gallagher-synonymous—or vice-versa? If there *is* such a pair, then the two definitions are *not* equivalent; if there is no such pair, then the two definitions *are* equivalent.

The assignment: prove (informally) or disprove (by finding a counter-example) that these two definitions are equivalent.

Due by: I will accept submissions until the final exam (May 9).